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A mesomechanical approach to inhomogeneous particulate composite undergoing localized damage: part II—theory and application

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Abstract

A mesomechanical framework for localized damage law in inhomogeneous particulate-reinforced metal matrix composites (PMMCs) is developed through a mesodomain simulation methodology in Part I. The average local stress-strain of mesodomains is derived by Eshelby's strain transformation method. The local damage rate in mesodomains is expressed in terms of local strain energy density of the subdomains. The overall weighting of local damage is then obtained by taking its spatial average over the entire domain. A finite element model of mesodomains undergoing localized damage reveals that the global damage factor is proportional to the local damage and the mesodomain's volume fraction. This holds until the plastic strain spreads into a macroscopic scale. An application of this mesomechanical damage theory is presented to predict the localized damage life of PMMCs.

1. Introduction

Experimental investigation has identified that particle fracture, debonding and short cracks are typically localized in the clustering regions in PMMCs (Li and Ellyin, 1996). This damage localization considerably reduces ductility and fatigue life of PMMCs, and remains an industrial concern for the application of particle- and whisker-reinforced composites.

Considerable effort has been devoted to predict the growth of short cracks and/or small cracks (mostly referred to as the three-dimensional short cracks, such as corner cracks and surface cracks) in the past two decades (Ellyin, 1997; Pearson, 1975). Due to the local microstructure-dependence, short crack growth behaviour cannot be described by linear elastic fracture mechanics (Ritchie and Lankford, 1986). Instead of attempting a correlation with the dramatically varying short crack growth rate, a

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damage theory for homogeneous materials has been developed to predict the life of short crack initiation and growth (Chaboche, 1988). In PMMCs a short crack grows much longer and is much more microstructure-sensitive than in metals and alloys (Li and Ellyin, 1996). The local material properties of clusters in PMMCs are significantly different from the surrounding domains. This has made the attempt to directly apply continuum damage mechanics to PMMCs ineffective.

To avoid the inherent ambiguities in macro-modelling, local approaches have been advocated. For example, Lemaitre's (1986) local approach assumes local yield stress degradation to fatigue limit. Damage localization in concrete is dealt with by a so-called non-local continuum with local strain (Bazant, 1990). Ductile and creep fractures have been successfully predicted by a local physical rupture criterion incorporated with local damage process (Rousselier et al., 1985). Finite element modelling has proven effective for local analysis, such as local damage in a concrete material (Lemaitre, 1986), local deformation in PMMCs (McHugh et al., 1993), short crack trapping in PMMCs (Li and Ellyin, 1994) and the micro-macro damage correlation investigation on interface debonding and particle fracture in PMMCs (Li and Ellyin, 1997).

Experimental evidence indicates that damage localization in PMMCs is strongly dependent on the local particle morphology, especially the local particle volume fraction in clusters and clustering distribution (Corbin and Wilkinson, 1994). High local volume fractions of particles results in considerably different local material properties in the cluster regions of the composites. A local approach incorporating local material properties would thus appear to be necessary. The idealized transformation of the special clustering regions into homogeneous mesodomains in a PMMC simplifies the particle distribution pattern, emphasizing the local average features of clustering regions of the composites.

Mesomechanics was proposed as a combination of microstructure and mechanics (Haritos et al., 1988). Li (1990) pointed out that this approach is suitable to deal with microstructure-sensitive behaviour, such as short crack growth. Here the word 'meso' not only means a combination of material structure-mechanics, but also a local scale, between the macro- and micro-scales.

The aim of this paper is to quantitatively evaluate the local stress–strain, to develop the local damage law in the clustering regions of PMMCs, and to predict the life of saturated local damage of clustering PMMCs by means of measurable mechanics and meso-structure parameters. This local analysis is based on the local homogenization of specific clustering regions using mesodomains (Part I). Eshelby's transformation strain method (Eshelby, 1957) is then applied to calculate the local average stress–strain in the mesodomains of an elastic composite. The local damage law inside the mesodomains is derived from the local parameters of the mesodomains and the global response to local damage is obtained by a spatial integration over the whole domain. The local stress–strain and the global response to local damage in the mesodomains undergoing progressive local damage are also analysed by a finite element mesodomain model in this study. Local damage development in mesodomains under a constant cyclic stress amplitude is represented by a decrease of the local elastic modulus of the subdomain medium. The application of this mesomechanical damage theory in prediction of short crack growth life is also discussed.

2. Local stress–strain in mesodomains

Eshelby (1957) analysed the local strain in inclusions in a heterogeneous material by replacing the inclusions with a transformation strain in the inclusion sites in a homogeneous medium. This method has been widely used for analysis of the local stress–strain field near inclusions (Mura, 1982), dislocations (Kroner, 1958), and cracks (Fu and Keer, 1969). The idealization of clustering structures into mesodomains makes it possible to apply Eshelby's method for a local stress–strain analysis in clustering regions of PMMCs.

As mentioned in Part I, the mesodomain size, l_{ms}, is determined by the transition condition from the localized damage stage to the damage globalization stage. The mesodomains are positioned in clustering regions of size equal to l_{ms} and with the highest effective local particle volume fraction among all regions of the same size. This will uniquely define a mesodomain for the clustering region with the highest prospect of localized damage, although the contours of $f^{v}_{loc,eff}$ may be interconnected in that region. The PMMC is then treated as a homogeneous two-phase material of mesodomains and 'matrix'. The volume fractions of mesodomains and 'matrix' are P_{ms} and $1 - P_{ms}$, respectively (Part I). This treatment does not change the overall constitutive relationship, but results in two homogeneous regions with known $f^{v}_{loc,eff}$ and f^{v}_{mt}.

Assuming that mesodomains are spherical in shape, and that their distribution results in overall isotropic properties, the local stress and strain in the mesodomains can be obtained in the simplest form. Fig. 1 displays a typical mesodomains pattern generated based on the above assumptions. For the case shown in Fig. 1, the nominal applied stress at the far field of the composite, σ^{o}_{ij} causes nominal strain ε^{o}_{ij}. The mesodomains with an elastic modulus, C^{ms}_{ijkl}, will have a strain disturbance ε_{ij}, and the stress disturbance inside the mesodomains due to the higher stiffness will be σ_{ij}. Applying Hooke's law for the local stress–strain in the mesodomains gives

$$\sigma^{o}_{ij} + \sigma_{ij} = C^{ms}_{ijkl}\left(\varepsilon^{o}_{kl} + \varepsilon_{kl}\right) \tag{1}$$

where the superscript o indicates the parameters as macro or nominal.

Now, assuming the composite is homogeneous everywhere with an average elastic modulus, C_{ijkl}, and a transformation strain, ε^{*}_{ij}, at the original locations of mesodomains, the ε^{*}_{ij} represents the different strain due to removal of the mesodomains (see the upper right region in Fig. 1). Therefore, the stress–strain for the same subdomains is given by,

$$\sigma^{o}_{ij} + \sigma_{ij} = C_{ijkl}\left(\varepsilon^{o}_{kl} + \varepsilon_{kl} - \varepsilon^{*}_{kl}\right) \tag{2}$$

The transformation strain ε^{*}_{ij} is zero outside the subdomains. Noting the transformation strain ε^{*}_{ij} is a fictitious one, equivalency of the above two expressions for stress–strain inside the same subdomains leads to

$$C^{ms}_{ijkl}\left(\varepsilon^{o}_{kl} + S_{klmn}\varepsilon^{*}_{mn}\right) = C_{ijkl}\left(\varepsilon^{o}_{kl} + S_{klmn}\varepsilon^{*}_{mn} - \varepsilon^{*}_{kj}\right) \tag{3}$$

in which S_{klmn} is the Eshelby's tensor. Eshelby's tensor depends on the shape of mesodomains, and has the simplest form for spherical inclusions. As both mesodomains and 'matrix' are isotropic, eqn (3) reduces to

$$\mu^{*}\left(\varepsilon^{o}_{ij} + \varepsilon_{ij}\right) + \lambda^{*}\delta_{ij}\left(\varepsilon^{o}_{kk} + \varepsilon_{kk}\right) = 2\mu\left(\varepsilon^{o}_{ij} + \varepsilon_{ij} - \varepsilon^{*}_{ij}\right) + \lambda\delta^{*}_{ij}\left(\varepsilon^{o}_{kk} + \varepsilon_{kk} - \varepsilon^{*}_{kk}\right) \tag{4}$$

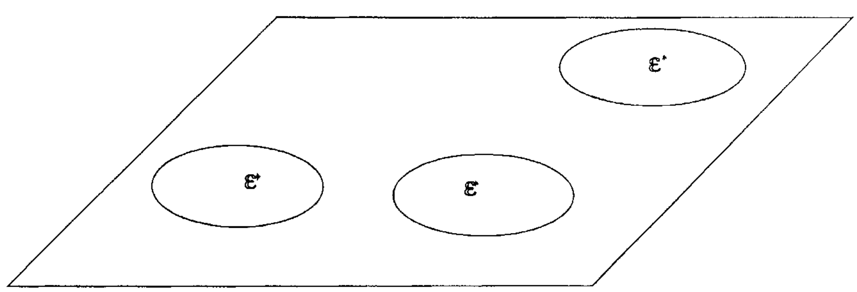

Fig. 1. Transformation strain in the original mesosomain location.

Once the strain is determined, the stress field inside the mesodomains can be obtained. The elastic strain in eqn (2) is related to stress σ_{ij} by Hooke's law:

$$\sigma^{\mathrm{ms}} = \sigma^{\mathrm{o}}_{ij} + \sigma_{ij} = 2\mu\left(\varepsilon^{\mathrm{o}}_{ij} - S_{ij\mathrm{mn}}\varepsilon^{*}_{\mathrm{mn}} - \varepsilon^{*}_{ij}\right) + \lambda\delta_{ij}\left(\varepsilon^{\mathrm{o}}_{kk} + S_{kk\mathrm{mn}}\varepsilon^{*}_{\mathrm{mn}} - \varepsilon^{*}_{kk}\right) \tag{5}$$

In the case of an applied pure shear stress, say σ^{o}_{12}, the above equation simplifies to,

$$\mu_{\mathrm{g}}\left(\varepsilon^{\mathrm{o}}_{12} + 2S_{1212}\varepsilon^{*}_{12} - \varepsilon^{*}_{12}\right) = \mu_{\mathrm{ms}}\left(\varepsilon^{\mathrm{o}}_{12} + 2S_{1212}\varepsilon^{*}_{12}\right) \tag{6}$$

in which subscript g indicates the material modulus as a global one for the solid. The local shear stress $\sigma^{\mathrm{ms}}_{12}$ in the mesodomain is then,

$$\sigma^{\mathrm{ms}}_{12} = \sigma^{\mathrm{o}}_{12} + \sigma_{12} = \frac{2\mu_{\mathrm{ms}}\mu_{\mathrm{g}}\varepsilon^{\mathrm{o}}_{12}}{\mu_{\mathrm{g}} + \dfrac{2(4-5v_{\mathrm{g}})(\mu_{\mathrm{ms}} - \mu_{\mathrm{g}})}{15\left(1-v_{\mathrm{g}}\right)}} \tag{7}$$

This equation shows that the local stress–strain in the mesodomain only depends on the difference of elastic moduli of mesodomain and 'matrix', or in other words, the local effective volume fraction of mesodomains.

The above analysis is based on the infinite body assumption. The effect of a finite body can be evaluated by an image stress method (Mura, 1982). It has shown that the finite body result is a fraction of the infinite body assumption, the value of which is the 'matrix' volume fraction, $1 - p^{\mathrm{v}}_{\mathrm{ms}}$.

The elastic stress concentration due to the high local volume fraction of particles is termed $K^{\mathrm{m}}_{\mathrm{t}}$ (Li and Ellyin, 1995). The $K^{\mathrm{m}}_{\mathrm{t}}$ is the ratio of the local average stress in the mesodomains over the overall average stress. For a PMMC with $v = 0.33$, $K^{\mathrm{m}}_{\mathrm{t}}$ approximates to

$$K^{\mathrm{m}}_{\mathrm{t}} = \frac{2.12}{1 + 1.12\dfrac{E_{\mathrm{g}}}{E_{\mathrm{ms}}}} \tag{8}$$

The above has been applied to evaluate short crack initiation (Li and Ellyin, 1995). The mesodomain composite elastic modulus E_{g} is the same as the modulus of the PMMC of the same loading history, and is therefore a measurable constant. E_{g} also can be evaluated by the self-consistent method (Hill, 1965). The effect of the mesodomain volume fraction on the local stress and stress concentration of the composite can thus be expressed.

3. Mesomechanical analysis of localized damage

The influence of localized damage in mesodomains on the overall composite can be obtained through spatial averaging (Bazant and Pijaudier-Cabot, 1988) as follows:

$$D_{\mathrm{g}} = \frac{1}{V}\int_{v} \alpha'(\boldsymbol{s}) D_{\mathrm{ms}}(\boldsymbol{s})\, \mathrm{d}V(\boldsymbol{s}) \tag{9}$$

in which $\boldsymbol{s}$ is the coordinate vector and α' is a weighting function having a value of $\alpha' = 1$ within mesodomains and $\alpha' = 0$ outside the subdomains. D_{ms} is assumed to be a constant across all the mesodomains under a certain applied stress level. Therefore, the integration of eqn (9) can be written as

$$D_{\mathrm{g}} = p_{\mathrm{ms}} D_{\mathrm{ms}} \tag{10}$$

Equations (9) and (10) provide a correlation between the local damage parameter at mesodomains with the global parameter. In other words, the local damage factor, D_{ms}, can be evaluated by measuring D_g and the volume fraction p_{ms} of mesodomains.

A damage factor, D_g, has been proposed which can be measured through the variation of elastic moduli (Lemaitre, 1985), i.e.

$$D_g = 1 - \frac{E_g}{E_{go}} \tag{11}$$

in which E_{go} is the elastic modulus of the material before any loading. As damage is localized in the mesodomains of PMMCs in the early stage, the local damage parameter, D_{ms}, indicates the deterioration of the local elastic modulus of mesodomains such that

$$D_{ms} = 1 - \frac{E_{ms}}{E_{mso}} \tag{12}$$

Substitution of the above into eqn (10), yields

$$D_g = p_{ms}\left(1 - \frac{E_{ms}}{E_{mso}}\right) \tag{13}$$

The overall elastic modulus at the time of local damage saturation, or the formation of a long crack, can be obtained by substituting $D_{ms} = 1$ into eqns (10) and (11) as

$$E_g = (1 - p_{ms})E_{go} \tag{14}$$

Therefore, the global elastic modulus signals the transition from localized damage stage to the damage globalization stage.

Based on a thermodynamics analysis, the elastic strain energy release rate is a driving force of the damage growth at constant stress and constant temperature (Chaboche, 1977; Krajcinovic, 1984). Applying this concept into mesodomains, the localized damage driving force, Y_{ms}, can be defined by the local elastic strain energy release rate, W^{ms}, due to local damage variation,

$$-Y_{ms} = \frac{W^{e}_{ms}}{1 - D_{ms}} \tag{15}$$

Y_{ms} can also be expressed as a function of hydrostatic stress, σ_h, and von Mises equivalent stress, σ_{eq}, as follows

$$-Y = \frac{\left(\sigma^{ms}_{eq}\right)^2}{2E_{ms}(1 - D_{ms})^2}\left[\frac{2}{3}(1 + v) + 3(1 - 2v)\left(\frac{\sigma^{ms}_{h}}{\sigma^{ms}_{eq}}\right)^2\right] \tag{16}$$

In the damage localization stage, the localized damage is the only cause of strain energy density degradation; it turns out that

$$W^{e} = p_{ms} W^{e}_{ms} \tag{17}$$

Substituting eqn (17) into eqn (15), one gets

$$-Y_{ms} = \frac{W^{e}}{P_{ms} - D_g} \tag{18}$$

This equation, similar to eqn (13), indicates the possibility of determining the local damage driving force by using measurable global parameters.

4. Finite element mesodomain modelling

4.1. On the models

In this study two types of finite element models, an embodied cluster model and a mesodomain model, were constructed for a 20% Al_2O_3 particulate-reinforced 6061 aluminum composite in the T6 heat-treatment condition. This FE analysis is used to evaluate the applied average stress–local strain in

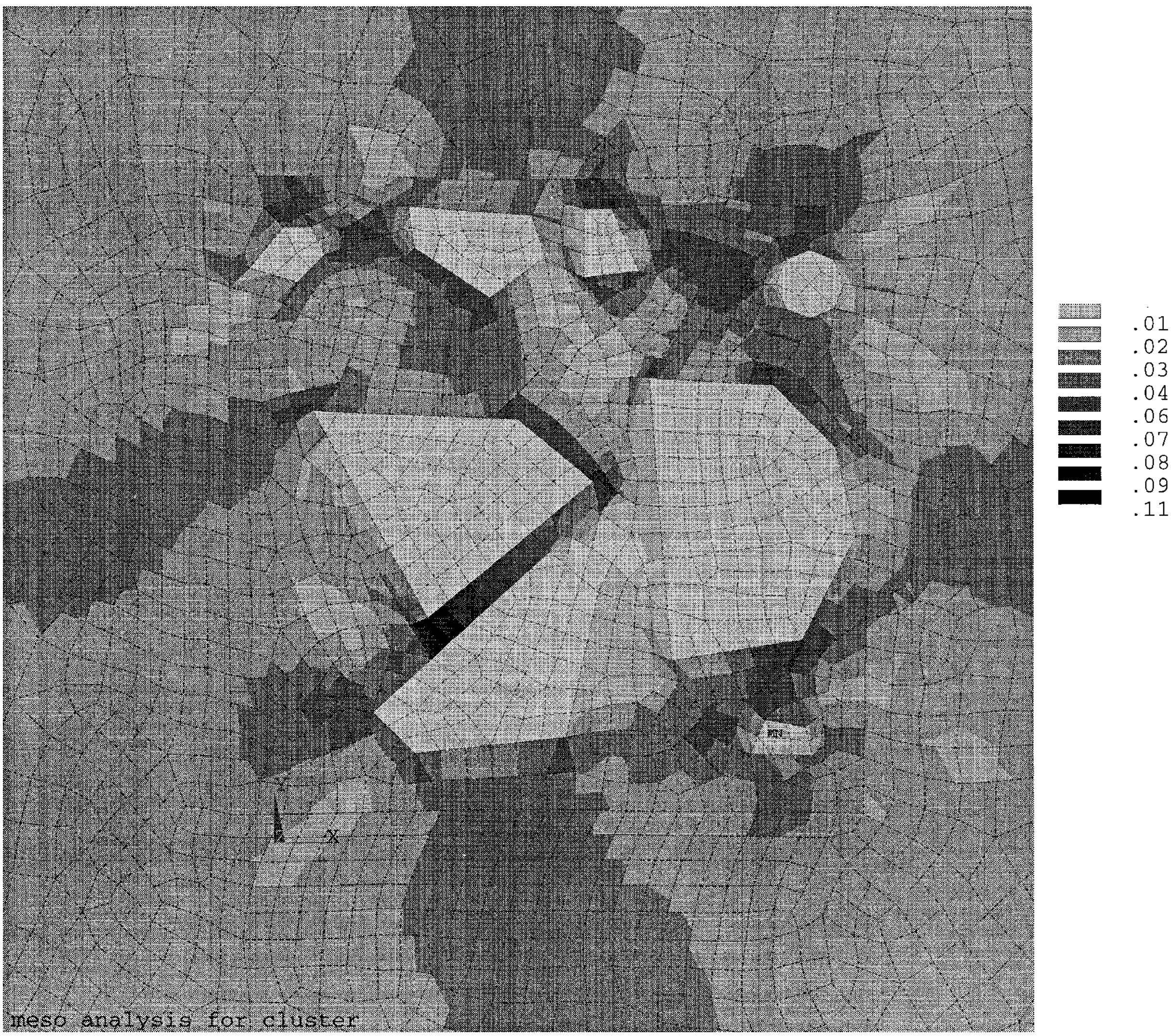

Fig. 2. The equivalent strain distribution under an applied stress of 200 MPa in cluster region by an embodied cluster finite element model.

the specific clustering regions which will accommodate the saturated local damage. The effect of progressive local damage on the local stress–strain in the cluster regions and the global damage factor can thus be investigated.

The embodied cluster model studied is for the cluster shown in Fig. 5 of Part I. The material properties of alumina and 6061 aluminum alloy at T6 condition were input for the particles and matrix in the cluster region (Li and Ellyin, 1994). The local particle volume fraction is 55% in the cluster area. The mesh surrounding the cluster is 12.5 times larger than the cluster area, therefore it represents a composite of 8% volume fraction clusters. The material properties of the composite were used for the surrounding mesh. Fig. 2 schematically displays a part of the finite element mesh around the cluster region with equivalent strain distribution under an applied stress of 200 MPa.

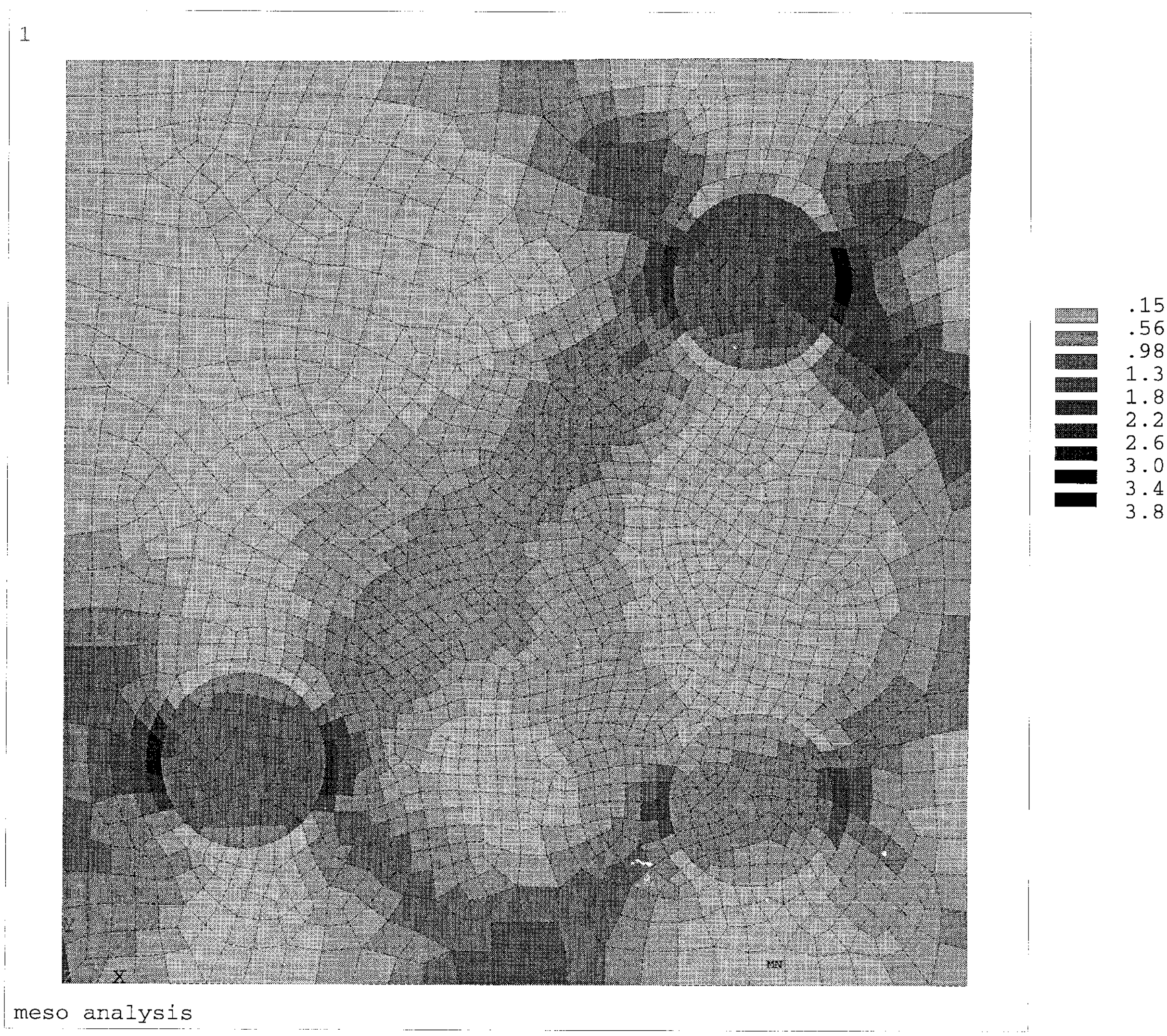

Fig. 3. Finite element mesh of a mesodomain model for a representative volume element of an inhomogeneous PMMC, with strain energy distribution at $D_{ms} = 0.77$.

The mesodomain model shown in Fig. 3 is for the same clustering composite (20% Al_2O_3/6061 Al). The model was based on a typical cluster distribution in the composite as shown in Fig. 1. The stress-flaw diagram for the composite was applied to determine the mesodomain size, 80 μm for the composite under the specified applied stress amplitude of 250 MPa. Figure 3 shows the mesodomain model with total strain energy density distribution at $D_{ms} = 0.77$ under a 250 MPa stress state. The mesodomain's effective particle volume fraction was evaluated as 55% by eqn (1) of Part I, while that of the 'matrix' was 17%. The mesodomains are 80 μm in diameter, while the whole mesh is 1000 μm in width and length. The average distance of the mesodomain centres is 600 μm. The volume fraction of mesodomains, P_{ms}, is 8%. Hence, this model represents a PMMC of the same volume fraction of clusters as the embodied model.

The material properties for the mesodomains and the 'matrix' were extrapolated from the experimental data obtained for Al_2O_3/6061 aluminum composites of different particle volume fraction. The elastic modulus for the mesodomains is 150,000 MPa, while that of 'matrix' is 89,000 MPa. The mesodomain model has been used to calculate the applied stress–local strain curve and evaluate the progressive local damage effects. For the latter, the progressive localized damage under a constant cyclic loading was incorporated by decreasing the elastic modulus of mesodomains step by step, while the matrix modulus was kept constant.

4.2. Comparison of mesodomain model with embodied cluster model

Figure 4 shows applied stress–local average strain curves obtained using the finite element models. The data of nominal applied stress–local strain by the embodied model are shown in the figure as solid circles, and the nominal stress–local strain by the mesodomain as solid squares. The average local strain was calculated by the average displacements of the nodes on the top boundary of the embodied cluster

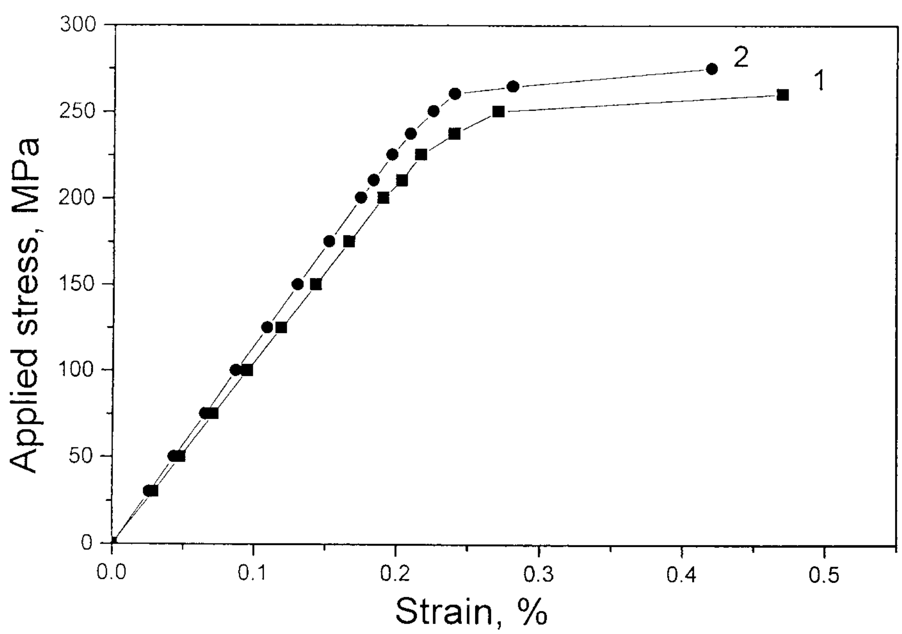

Fig. 4. Comparison of applied stress–average local strain curves by the embodied cluster model and the mesodomain model.

model. The local strain displayed for the mesodomains was taken as the averaged value from all elements inside the subdomains. All the above cases are shown vs the applied stress in Fig. 4.

It is interesting to note that the mesodomain model displays slightly lower applied stress–local strain curve than the embodied cluster model. The higher local stress in the embodied cluster of the same average local strain is related to the large particles and sharp angle of the particles in the cluster. However, the basic agreement between the two models supports the mesodomain approach for idealizing clustering composites. On the other hand, the finite element meshing of the mesodomain model is simpler. Therefore, a much larger volume, or a representative volume of a PMMC with a number of clusters, can be modelled in this way. Thus it can reveal the effect of cluster distribution, i.e. the 'outer' effect of clusters on local stress–strain in the clusters.

4.3. Effect of progressive localized damage in mesodomains

The effect of progressive localized damage on the local and global parameters of a PMMC under fatigue loading was evaluated using the mesodomain model. Figure 5 displays the variation of global elastic modulus, E_g, and local average stress in the mesodomains as the local damage gradually increases under a constant applied stress amplitude of 250 MPa. All data are expressed in dimensionless parameters, i.e., ratios of a parameter at a damage state to the value of the same parameter prior to damage-initiation.

As expected, the global elastic modulus decreases, or the global damage factor increases as the progressive local damage, D_{ms}, increases in the earlier stage of cyclic loading. The global elastic modulus variation is almost linearly proportional to the local one in the range of $D_{ms} < 0.75$. This accords with eqn (12) which indicates a linear relationship between E_{ms} and D_g. A dramatic drop of the global elastic modulus takes place when $D_{ms} = 0.75$, which results from plastic deformation inducing the 'load'

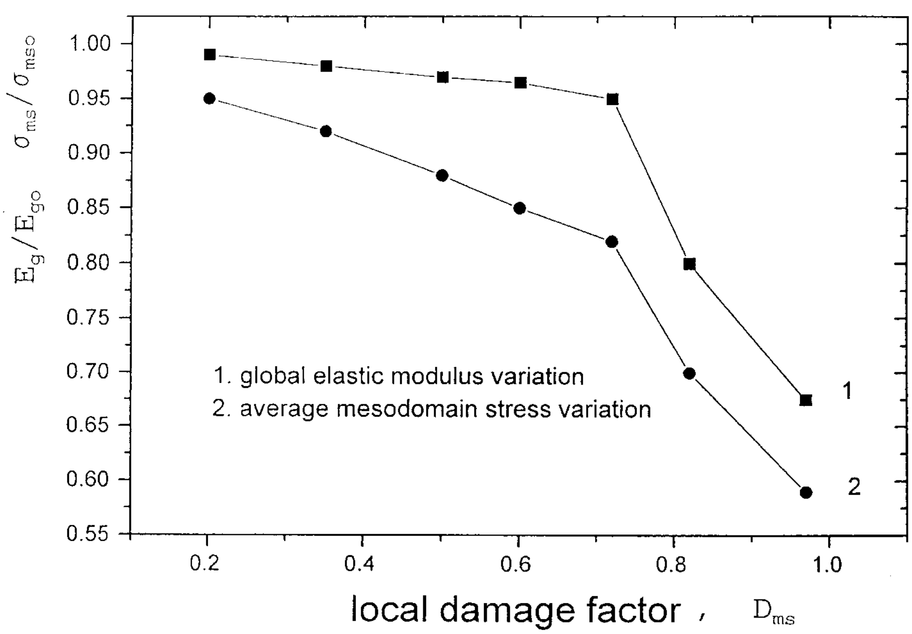

Fig. 5. Variation of global elastic modulus, average local stress in the mesodomains undergoing progressive local damage under a constant cyclic stress amplitude, 250 MPa.

transfer from the deteriorated mesodomains to the 'matrix'. As shown in Fig. 3, the maximum strain energy density is located at an area near the mesodomains when $D_{ms} = 0.77$. This drastic change shows that damage globalization can occur much earlier than $D_{ms} = 1$. In the range of $D_{ms} > 0.75$, the weight function in eqn (9) should be different due to the plastic deformation effect. This figure also shows that the local stress in the mesodomains decreases as local damage increases. The local stress decreases much faster with the increase of local damage in the range of $D_{ms} > 0.75$.

At a much lower applied cyclic stress amplitude, the local damage may be fully developed until $D_{ms} = 1$ without inducing plastic deformation in the nearby 'matrix'. As the finite element results indicated, D_g is approximately equal to P_{ms} at $D_{ms} = 1$, which is predicted by eqn (13).

The mesodomain model can also reveal the effect of cluster distribution and cluster volume fraction on the local parameters in a clustering region and the global damage factor. Generally, a close mesodomain distribution enhances the local stress–strain in the mesodomains and in the 'matrix' areas between the subdomains. As a result, close mesodomain distribution promotes damage globalization.

5. Application to short crack problems in PMMCs

Growth rate of short (small) cracks is microstructure-sensitive, and cannot be described by linear elastic fracture mechanics (Ritchie and Lankford, 1986). A main objective of continuum damage mechanics is to predict the 'life time' of structures, which includes the short crack initiation and growth (Chaboche, 1988). It has been shown that short crack growth in PMMCs under a constant cyclic loading varies in both direction and growth rate in a much more dramatic fashion than in metals and alloys (Li and Ellyin, 1995). Moreover, significant material inhomogeneity in PMMCs prevents a direct application of continuum damage mechanics in these materials.

As mentioned in Part I, the failure process of PMMCs consists of two stages: a damage localization stage and a damage globalization stage. The latter is a long crack propagation process which closely follows Paris law in the intermediate stress intensity factor range. The damage localization process includes particle interface debonding, particle fracture, short crack initiation and propagation in clustering regions of PMMCs. There is no clear size definition of short crack initiation, nor of long crack initiation. In this study the long crack initiation refers to a crack of a minimum length that begins to follow the Paris law at the specified stress level. In this way, the saturated local damage life corresponds to the initiation life of a long crack.

The total fatigue life of a PMMC component consists of the saturated local damage life, N_{li}, and long crack growth life, N_{lp}, i.e.,

$$N_f = N_{li} + N_{lp} \tag{19}$$

N_{li} represents the fatigue life when $D_{ms} = 1$. As mentioned in Part I, local damage saturation is correlated with the global damage parameter,

$$D_g = P_{ms}, \quad D_{ms} = 1; \quad N = N_{li} \tag{20}$$

The strain energy density has been shown to be a suitable parameter for both uniaxial and multiaxial cyclic loading (Ellyin, 1997). This parameter can also be applied to the mesodomains. Local damage growth rate can be expressed in terms of local average strain energy density in the mesodomain, ΔW_{ms}, as

$$\frac{dD_{ms}}{dN} = D_{ms}^{a} \Delta W_{ms}^{b} \tag{21}$$

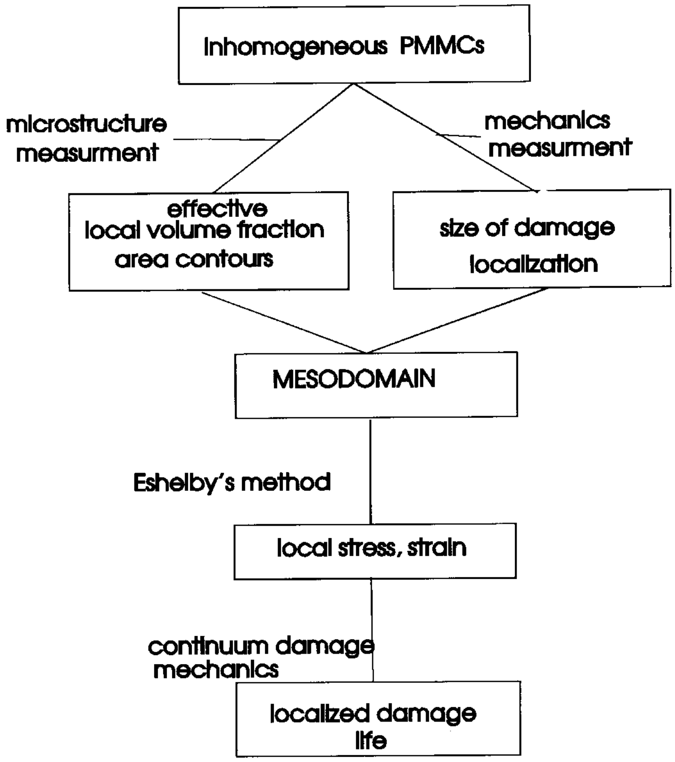

Fig. 6. A flow chart of localized damage life prediction by the mesomechanical damage approach.

in which a and b are material constants, and ΔW_{ms} is the sum of plastic strain energy $\Delta W^{\mathrm{p}}_{\mathrm{ms}}$ and the tensile elastic strain energy, $\Delta W^{\mathrm{e+}}_{\mathrm{ms}}$ (Ellyin, 1997). Similarly, the saturated local damage life can be obtained by integrating the above equation from $D_{\mathrm{ms}} = 0$ to $D_{\mathrm{ms}} = 1$. This results in

$$N_{1i} = \frac{1}{(1-a)\Delta W^{b}_{\mathrm{ms}}} \tag{22}$$

The above process can be calculated in a computer program and a typical flow chart is shown in Fig. 6. First, through combined microstructure and mechanics measurements for an inhomogeneous PMMC, the size and the effective local particle volume fraction of mesodomains can be determined. Then the local strain and strain energy inside the mesodomain can be obtained by applying the transformation strain method, while the long crack initiation life can be predicted by applying the damage law. Long crack growth in PMMCs tends to follow the Paris law or a modified version of it, and can be predicted using well-established methods in engineering design (Ellyin, 1997). In this way the entire life of an engineering structure made of the PMMC can be predicted.

6. Summary

Transformation of clustering PMMCs into homogeneous mesodomains and 'matrix' enables derivation of local average stress–strain and local damage law in the clustering regions.

The elastic local stress–strain of mesodomains of a spherical shape is derived by using Eshelby's transformation strain method. It is shown that the local stress concentration in the mesodomains is proportional to their local effective volume fraction of particles.

A local damage law in the clustering regions of a PMMC is obtained by incorporating continuum damage mechanics with the local stress–strain and local material properties of the mesodomains. The overall appearance of the local damage in mesodomains is determined by the volume fraction of mesodomains. As a result, the life of local damage saturation, or long crack initiation, can be evaluated through the global stiffness measurement.

The applied stress–local average strain curves calculated by a finite element mesodomain model and an embodied cluster model for an inhomogeneous alumina reinforced 6061 aluminum alloy composite concur. The finite element mesodomain results of local average stress decrease with local damage development agree well with the results found using Eshelby's (1957) method only when the local damage factor is not larger than a certain amount ($D_{ms} < 0.75$). The dramatic increase of global damage factor when local damage factor $D_{ms} > 0.75$ is attributed to the spreading of plastic deformation into a relatively large scale of 'matrix', which assists the release of local damage.

Mesomechanical damage theory effectively simplifies and enhances prediction of the short crack initiation and propagation in PMMCs that result in saturated local damage. This methodology circumvents industrial concerns and academic difficulties related to the dramatically variable short crack growth rates.

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